LiDAR Feature Outlier Mitigation Aided by Graduated Non-convexity Relaxation for Safety-critical Localization in Urban Canyons

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***Abstract*—Safety-critical localization is essential for unmanned autonomous systems. LiDAR localization gains great popularity in urban canyons due to its high ranging accuracy. Inheriting from the integrity monitoring theory for GNSS, safety-certifiable LiDAR localization first consists in fault detection and exclusion (FDE). In face of numerous LiDAR measurements, conventional chi-square test for FDE is computationally intractable. What's more, inliers could be mistakenly excluded without reconsideration. This paper proposes a computationally tractable and flexible FDE method. It's realized via outlier mitigation aided by graduated non-convexity (GNC) relaxation. The two novel loss functions truncated least square (TLS) and the Geman McClure (GM) are combined respectively. The outlier-mitigated planar-feature-based LiDAR localization is formulated with GNC and TLS or GM. More importantly, a triple-layer optimization method is proposed to solve the localization formulation. Besides the typical GNC relaxation, the control parameter is taken into consideration for tuning the outliers resistance degree. The outlier mitigated pose estimation and the weightings ranging from 0 to 1 for the exploited LiDAR measurements are finally produced. Extensive experiments of the proposed method is conducted on urban dataset. What's more, considering that TSL and GM provides distinct outlier mitigation patterns, the performances from them are investigated and compared.**

Keywords—3D LiDAR, satety-critical localization, urban canyons, fault detection and exclusion, graduated non-convexity relaxation

I. Introduction

Reliable and accurate localization is of great importance for the realization of unmanned autonomous systems [1]. The last decade has witnessed a substantial breakthrough in the accuracy of localization [2]. Nevertheless, safety certification is the prerequisite for the large-scale deployment of safety-critical applications such as autonomous driving vehicles [3]. In particular, the safety-certifiable localization emphasizes that the potentially largest localization error should be quantified and the alarm should be triggered when the potential error exceeds the maximum acceptable error. Integrity is developed as a safety certification of the global navigation satellite system (GNSS) in the aviation field [4]. Specifically, integrity monitoring is conducted by two steps: (1) the fault (namely outlier) detection and exclusion (FDE) which is typically performed via the Chi-square test. The statistics of the residuals generated by the measurement model are subjected to Chi-square distribution. Measurement with the largest residual is detected and excluded as fault successively until the Chi-squared test is passed; (2) the calculation of the protection level (PL) which is defined as the potentially largest localization error (yet not in the scope of this paper). Therefore, a reliable FDE is of great importance for integrity monitoring.

GNSS signals suffer from reflections and blockages leading to multipath effects and non-line-of-sight (NLOS) receptions in urban canyons. As a result, the positioning error is significantly degraded which can easily reach 10 meters in dense urban scenarios [5]. The light detection and ranging (LiDAR) based localization exploiting prior 3D point cloud maps has recently gained lots of attention which can provide centimeter-level accuracy by matching the real-time 3D point clouds with the prior 3D point cloud map [6]. However, as shown in our recent evaluation of diverse LiDAR localization schemes in urban canyons [7], outliers such as that from dynamic objects bring new challenges to the existing LiDAR localization. Therefore, to certify the safety of the LiDAR localization solution, a reliable FDE of the LiDAR measurements is highly desirable which is similar to the GNSS integrity monitoring. Differently, the number of measurements involved in the LiDAR localization problem is significantly more than the ones from the GNSS (e.g. 1,000 LiDAR features for the LiDAR localization problem). As a result, a greedy search-based Chi-square test [8] for the FDE of LiDAR measurements would lead to an unacceptable computational load. In addition, the conventional Chi-square test-based FDE excludes the measurements one-by-one which could lead to the mis-exclusion. The excluded measurements would not be re-considered in the next iteration.

Recently, the work [9] proposed employing the Black-Rangarajan duality [10] and the graduated non-convexity (GNC) relaxation to mitigate the impacts of the outlier measurements in the point cloud registration problem. In particular, the pose of the system and the weightings of the measurements are estimated simultaneously by using the novel non-convex loss function, such as the Geman McClure (GM) and Truncated Least Square (TLS) functions. By combining the GNC and TLS, the estimated weightings of the measurements are either 0 or 1 with a binary option. In particular, the measurement is excluded from the localization if the weighting is 0. By combining the GNC and GM, the estimated weightings of the measurements range from 0 to 1 which aims to mitigate the impacts of the outlier measurements. Different from the Chi-square test, the GNC detects the outliers by considering all the measurements simultaneously and the outlier detection could converge in several iterations regardless of the number of measurements. To solve the non-convexity issue introduced by the novel loss functions to the original objective function, surrogate function is defined in GNC. Under the control of a non-convexity relaxation parameter, the surrogate function recovers the original objective function gradually from an initial convex pattern. Consequently, GNC computes a solution to the non-convex problem, by starting from its convex surrogate and gradually increasing the amount of non-convexity by tuning the non-convexity relaxation parameter till the original objective function is recovered. As an extension, we implement the GNC to the GNSS outlier mitigation in [11] and improved positioning accuracy is achieved. However, the performance of the GNC in outlier mitigation relies on the selection of the kernel parameters of the applied non-convex loss function. In particular, the kernel parameter determines the non-convexity of the loss function. T-LOAM [12] combined the TLS and feature based-LiDAR localization to alleviate the impact of outliers. However, the performances of TLS and the softer loss function GM hasn't been investigated and compared.

To fill these gaps, this paper proposes a method to mitigate the LiDAR feature outliers by GNC combined with the Chi-square test targeted at safety-critical localization in urban canyons. The main innovations are summarized as follows, (1) Different from traditional GNC, the kernel parameter of the non-convex loss function deciding the mitigation performance is additionally relaxed. The Chi-square test is combined providing a quantitative guarantee of the outlier mitigation performance. (2) The outlier mitigation performance under the two loss functions, TLS and GM, providing different outlier mitigation patterns are extensively evaluated and discussed.

The paper is organized as follows. An overview of the proposed method is given in Section II. The detailed LiDAR localization method with outlier mitigation aided by GNC is introduced in Section III. Real-life experiments and discussions are presented in Section IV.

II. OVERVIEW

The overview of the proposed framework is shown in Fig. 1. The input is the point cloud at each epoch and the prior point cloud map. Planar features are extracted referring to the geometric smoothness of each point [13]. The LiDAR localization is based on planar feature registration between that from the current point cloud and that from the prior point cloud map. The registration is formulated as a non-linear optimization problem. Inspired by [9], the planar feature registration objective wrapped by the non-convex loss function is formulated as two terms. One is the weighted squared sum of the residuals between the feature and its corresponding plane [14]. The other is the penalty term derived from the robust kernel including the non-convexity relaxation parameter [10].

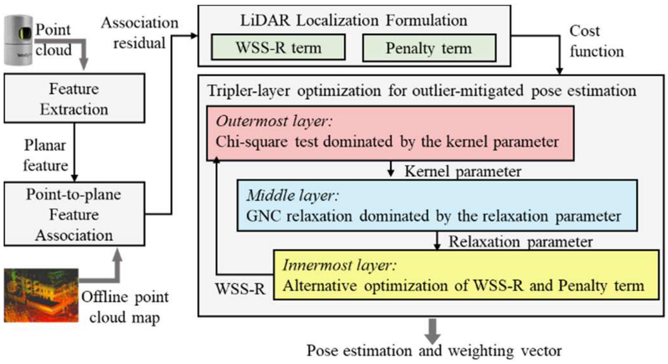

Fig. 1. Overview of the framework proposed in this paper.

Triple-layer iterated optimization guided by GNC is designed. The outermost layer is dominated by the kernel parameter of the applied non-convex loss function. The iteration on this layer is terminated until the Chi-square test on the converged cost is passed otherwise the kernel parameter would be decreased with the outlier mitigation degree increasing. The middle layer is dominated by the GNC non-convexity relaxation parameter which starts minimizing the convex surrogate of the re-formulated objective. The iteration on this layer is terminated until the non-convexity relaxation parameter goes to a limit (for example, goes below 1 given GM kernel) where the non-convex original objective is recovered. The innermost layer is dominated by the alternative update of the pose state and the weight for each residual. The former is performed with fixed weights and terminated until the weighted squared sum of the residuals (WSS-R) is converged. The latter is solved in closed form with fixed residuals in the penalty term. The output is the localization results after outlier mitigation.

In this paper, matrices are denoted as bold uppercase letters. Vectors are denoted with bold lowercase letters. Variable scalars are denoted with lowercase italic letters. Constant scalars are denoted with lowercase letters. In addition, the LiDAR body frame at the k epoch is represented as $\{\cdot\}^{\mathcal{L}_k}$. The map frame, defined as the initial LiDAR body frame, is kind of local world frame and denoted as $\{\cdot\}^{\mathcal{W}}$.

III. METHODOLOGY

This section present the details of the proposed LiDAR feature outlier mitigation method aided by GNC relaxation for safety-critical localization in urban canyons.

A. Planar feature-based LiDAR localization

The localization is implemented by the registration of the planar features from the current point cloud and the prior map. The registration is formulated as a non-linear optimization problem. The residual is defined as the distance between the plane point $\mathbf{p}_{k,i}^{\mathcal{L}_k}$ and its associated local plane as follows [14],

$$d_{k.i}(\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}}) = \mathbf{n}_{k,i}^T(\boldsymbol{\varphi}_{\mathcal{L}_k}^{\mathcal{W}} \otimes \mathbf{p}_{k,i}^{\mathcal{L}_k} + \mathbf{t}_{\mathcal{L}_k}^{\mathcal{W}}) + d_{k,i} \tag{1}$$

The 6d pose of the LiDAR at the current epoch is to be estimated and denoted as $\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}} = \{\boldsymbol{\varphi}_{\mathcal{L}_k}^{\mathcal{W}}, \mathbf{t}_{\mathcal{L}_k}^{\mathcal{W}}\} \in \mathbb{R}^6$. The rotation part is $\boldsymbol{\varphi}_{\mathcal{L}_k}^{\mathcal{W}} \in so(3)$ and the translation part is $\mathbf{t}_{\mathcal{L}_k}^{\mathcal{W}} \in \mathbb{R}^3$ [15]. "$\otimes$" denotes multiplication in $so(3)$to realize the rotation transformation. $\mathbf{n}_{k,i}^T$ and $d_{k,i}$ is the parameter of the associated local plane obtained via principal component analysis (PCA) of neighbors around $\mathbf{p}_{k,i}^{\mathcal{L}_k}$ in the map.

The cost function of the optimization wrapped by loss function for outlier mitigation is represented as,

$$f(\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}}) = \min_{\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}}} \sum_{i=1}^{N_{\mathcal{L}_k}} \rho(d_{k.i}(\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}})) \tag{2}$$

Where the $N_{\mathcal{L}_k}$ is the number of planar features. $\rho(\cdot)$ represents the loss function. The optimization is guided under a kind of maximum consensus. Intuitively, outlier makes up small proportion. When the residual is significantly large, the feature is assumed as outlier providing abnormal observation. The loss function mitigates the outliers via suppress the cost from these abnormal observations while maintain those small cost from large proportions of observations.

B. Planar feature-based LiDAR localization aided by GNC relaxation combined with TLS or GM

According to the Black-Rangarajan duality [10], the cost function wrapped by the loss function could be reconstructed as the addition of the WSS-R term and a penalty term,

$$f(\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}}, \boldsymbol{\omega}_k, {}^{\rho}c) = \min_{\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}}, \boldsymbol{\omega}_k} \sum_{i=1}^{N_{\mathcal{L}_k}} (\omega_{k.i} \left\| d_{k.i}(\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}}) \right\|^2 + \Pi_{{}^{\rho}c, {}^{\rho}r}(\omega_{k.i})) \tag{3}$$

Where $\omega_{k.i} \in [0,1]$ denotes the weighting for each planar feature. $\boldsymbol{\omega}_k$ denotes the weighting vector stacked by $\omega_{k.i}$ with $i = 1, \cdots, N_{\mathcal{L}_k}$. The WSS-R term is represented as $\sum_{i=1}^{N_{\mathcal{L}_k}} \omega_{k.i} \left\| d_{k.i}(\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}}) \right\|^2$. The penalty term of the weighting derived from the loss function is denoted as $\Pi_{{}^{\rho}c, {}^{\rho}r}(\omega_{k.i})$. Its controlled by the kernel parameter ${}^{\rho}c$ of the loss function and the relaxation parameter ${}^{\rho}r$. In the following, the superscript ${}^{\rho}(\cdot)$ will be replaced by the abbreviation of specific loss function. The ${}^{\rho}c$ controls the shape of the loss function which determines the resistance degree to the outliers. The ${}^{\rho}r$ controls the convexity of the cost function. Given initial value of ${}^{\rho}r$, the equation (3) is the convex surrogate of the original cost function. As the ${}^{\rho}r$ goes to its limit, the original non-convex cost function is recovered. The optimization is iterated under the control of ${}^{\rho}r$ and terminated until the original cost function is recovered. For each iteration, the initial guess for the minimization process of equation (3) is inherited from the last round. The GNC relaxation manner provides robust global estimation without requiring an initial guess in spite of the non-convexity.

The two novel loss function TLS and GM are exploited in this paper. As depicted in Fig.1 in [9], they provide two distinct outlier mitigation patterns respectively. TLS is defined as

$${}^{TLS}\rho(d) = \begin{cases} d^2 & , |d| \le |{}^{TLS}c| \\ {}^{TLS}c^2 & , |d| > |{}^{TLS}c| \end{cases} \tag{4}$$

The corresponding penalty term for the two loss function is chosen referring to [9] and [10] as follows,

$$\Pi_{{}^{TLS}c, {}^{TLS}r}(\omega_{k.i}) = \frac{{}^{TLS}r(1-\omega_{k.i})\,{}^{TLS}c^2}{{}^{TLS}r + \omega_{k.i}} \tag{5}$$

With TLS, the weightings for measurements producing costs larger than the kernel parameter ${}^{TLS}c$ are assigned as 0. Constraints from these measurements don't contribute to the optimization as the Jacobian matrix of the constant cost relative to the pose is zero. While, the weightings for measurements producing costs smaller than ${}^{TLS}c$ are assigned as 1. As defined in equation (4), the TLS maintains the cost as itself in this case. Thus, the penalty term should be canceled. As defined in equation (5), the weighting assigned with 1 reaches this goal. With the relaxation parameter ${}^{TLS}r$ goes from zero to positive infinity, the original non-convex cost function is recovered from its convex surrogate gradually.

The GM is defines as

$${}^{GM}\rho(d) = \frac{{}^{GM}c^2 d^2}{{}^{GM}c^2 + d^2} \tag{6}$$

With GM, each measurement is assigned a weighting ranging from 0 to 1. The large costs will be decreased while the small costs will be maintained to some extent. The penalty term $\Pi_{{}^{GM}c, {}^{GM}r}(\omega_{k.i})$ is chosen referring to [9] and [10] as follows,

$$\Pi_{{}^{GM}c, {}^{GM}r}(\omega_{k.i}) = {}^{GM}r\,{}^{GM}c^2\left(\sqrt{\omega_{k.i}} - 1\right)^2 \tag{7}$$

In contrast to TLS, with the relaxation parameter ${}^{GM}r$ goes from positive infinity to zero, the original non-convex cost function is recovered from its convex surrogate gradually.

As mentioned before, for both TLS and GM, the kernel parameter ${}^{TLS}c$ and ${}^{GM}c$ determines the resistance degree towards large costs. More importantly, smaller kernel parameter introduces stronger resistance. The large cost will be mapped to a smaller one.

C. Triple-layer Iterated Optimization for Outlier-mitigated LiDAR localization

The LiDAR localization is formulated according to the combination of GNC relaxation and TLS or GM as introduced in Section III.B. This section presents the details of the proposed triple-layer iterated optimization method of the formulation for outlier-mitigated LiDAR localization. As shown in Algorithm 1, Chi-square test is performed on the outermost layer based on the solution from the two inner layer iteration. Convexity relaxation is performed on the middle layer. Alternative optimization of the two decoupled terms in equation (3) is performed on the innermost layer. Typical LiDAR localization aided by GNC relaxation is dominated by the relaxation parameter only. Additionally, the proposed method take the kernel parameter of the loss function into consideration. Furthermore, a quantitative guarantee of the outlier mitigation performance is achieved via the Chi-square test. The outermost layer optimization is depended on the results from the inner layer. Thus the innermost layer is presented first.

Algorithm 1: The triple-layer optimization method for the outlier-mitigated LiDAR localization aided by GNC relaxation combined with TLS or GM

Outmost layer:
Inputs: The kernel parameter ${}^{\rho}c$, number of features $N_{\mathcal{L}_k}$, significance level α for Chi-squared test.
Do {
Middle layer:
  Inputs: The relaxation parameter ${}^{\rho}r$
  If $\rho(\cdot)$ is TLS,
    While ${}^{TLS}r \rightarrow +\infty$,
      Innermost layer: Algorithm 2
      GNC relaxation: ${}^{TLS}r \leftarrow 1.4 * {}^{TLS}r$
    End While
  Else if $\rho(\cdot)$ is GM,
    While ${}^{GM}r \rightarrow 0$,
      Innermost layer: Algorithm 2
      GNC relaxation: ${}^{GM}r \leftarrow {}^{GM}r/1.4$
    End While
}
Chi-square test:
While $\sum_{i=1}^{N_{\mathcal{L}_k}} \omega_{k.i}\left\|d_{k.i}\left(\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}}\right)\right\|^2 > \mathcal{X}^2_{N_{\mathcal{L}_k}-6,1-\alpha}$
  ${}^{\rho}c \leftarrow {}^{\rho}c/1.4$
End While
Outputs: The pose estimation result $\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}}$, the weightings for each measurements $\boldsymbol{\omega}_k$.

- The innermost layer.

On the innermost layer, the minimization of equation (3) is performed given determined kernel parameter and relaxation parameter set in the two outer layers. As shown in Algorithm 2, the optimization of the WSS-R and the penalty term is implemented in turn. The initialization for the optimization on this innermost layer is inherited from the last round middle layer optimization. However, at the first middle layer optimization, the weightings and the pose are initialized as 1 and identity respectively. There is no requirement for initial guess [10]. The pose sate optimization is performed first with fixed weighting vectors via the typical non-linear minimization algorithm Levenberg-Marquart [16]. The weighting vector is updated after then. With fixed pose states, closed-form solution of the weighting vector is derived from the penalty term. It's in the light of that the optimal solution is achieved when the gradient matrix of the penalty term relative to the weightings is zero.

Algorithm 2: Alternative optimization of the WSS-R and penalty term (The innermost layer optimization method in Algorithm 1)

Inputs: The kernel parameter ${}^{\rho}c$, the relaxation parameter ${}^{\rho}r$, weighting vector from the last round middle layer optimization $(\boldsymbol{\omega}_k)_m$, pose state from the last round middle layer optimization $\left(\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}}\right)_m$
Weighting Initialization: $\boldsymbol{\omega}_k \leftarrow (\boldsymbol{\omega}_k)_m$ from the last round middle layer optimization
Pose Initialization: $\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}} \leftarrow \left(\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}}\right)_m$ from the last round middle layer optimization
Do{
Pose state optimization with fixed weighting vector $\boldsymbol{\omega}_k$:
$\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}} \leftarrow \min_{\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}}} \sum_{i=1}^{N_{\mathcal{L}_k}} \omega_{k.i}\left\|d_{k.i}\left(\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}}\right)\right\|^2$
Weighting update with fixed pose state $\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}}$:
$\boldsymbol{\omega}_k \leftarrow \min_{\boldsymbol{\omega}_k} \sum_{i=1}^{N_{\mathcal{L}_k}} \Pi_{{}^{\rho}c,{}^{\rho}r}(\omega_{k.i})$
}
Output: updated $\mathbf{x}_{\mathcal{L}_k}^{\mathcal{W}}$ and $\boldsymbol{\omega}_k$

- The middle layer.

On the middle layer, the relaxation parameter of the loss function dominates the iteration. The non-convexity of the decoupled cost function (3) is gradually increased from its convex surrogate one with the changing of the relaxation parameter. For each determined relaxation parameter, the innermost layer optimization is triggered. When the relaxation parameter reaches one limit, the original non-convexity cost function is recovered and thus the middle layer optimization is terminated. For implementation, the threshold for the relaxation parameter is designed referring to [9].

- The outermost layer.

On the outermost layer, the kernel parameter of the loss function dominates the iteration. The noise of the registration is assumed to be zero-mean Gaussian distributed. Consequently, the WSS-R is subjected to Chi-square distribution. The Chi-square test is performed on the WSS-R regarding the solution produced by the two inner layer optimization process. With outliers reasonably mitigated, the WSS-R should be close to zero and pass the test. With outliers failed to be successfully mitigated, the WSS-R will exceeds the thresholds determined by the significance level of the test. The kernel parameter should be decreased in this case to provide stronger resistance towards the outliers. The optimization on the two inner layer will be executed once more until the test is passed.

IV. Experimental Results

The experiment is conducted in typical urban canyons. The 3D LiDAR Velodyne HDL-32E is exploited to collect 3D point clouds. The prior map is built in advance by registring the point clouds with the pose solutions from NovAtel Span-CPT, which also serves as the groud truth of the pose estimation. More details of the data collector setup could be found in our open-sourced dataset UrbanNav [5]. The data selected for evaluation of the proposed method is collected near Kowloon Tong in Hong Kong and abbreviated as KLT in the following.

The root mean square of absolute translation error (ATE) implemented by evo [17] is utilized to evaluated the pose estimation quantitatively. The relative translation increment is set to 1 meter by default. The performance of the following three LiDAR localization pipelines with differen outlier mitigation strategies are evaluated,

1) Pipeline1: The LiDAR localization is formulated as the squared sum of the feature registration residual defined as equation (1). Outlier mitigation is implemented iteratively via conventional Chi-square test.

2) Pipeline2: The LiDAR localization is formulated as the decoupled one defined as equation (3) with GNC and TLS or GM. Covential GNC is exploited without the adjustment of the kernel parameter and further Chi-square test.

3) Pipeline3: The LiDAR localization is formulated as the decoupled one defined as equation (3) with GNC and TLS or GM. The proposed tiple-layer optimization method is expolited to solve the pose.

The experiment results shows that for pipeline1, the RMS ATE is around 10%. While the RMS ATE for pipeline2 is around 9%. For pipeline3, the RMS ATE is around 7% and 8% with the loss function instantiated as TLS and GM respecively. The proposed method achieved the best accuracy. The effectiveness of the proposed method is validated. For the proposed method, slightly better performance is achieved with TLS compared with that with GM. The hard resistance to outliers of TLS outperforms the soft resistance manner of GM.

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